

Lec 10 digital Control

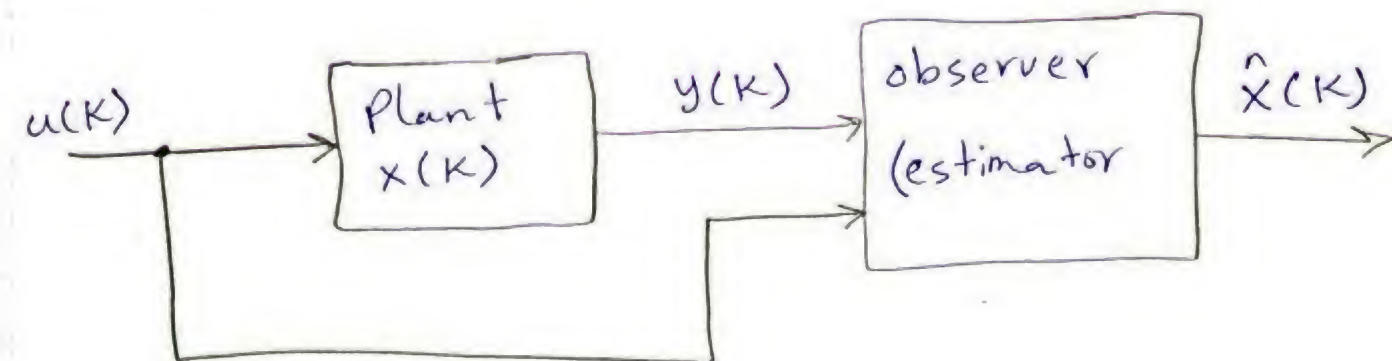
⇒ observer Design

→ For state Feedback Control (Pole Placement design)

$$u(k) = -K x(k)$$

→ It is required for all states to be accessible for measuring:

⇒ In this case if some or all of the states are not accessible for measuring, an observer should be designed to estimate these states values. The estimation is done by the history of data for i/p and o/p.



→ The observer equation:-

$$\hat{x}(k+1) = (A - Gc) \hat{x}(k) + Bu(k) + G y(k)$$

Where:

$G \rightarrow$ Gain matrix

$$\text{Gain matrix } G = \begin{pmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{pmatrix}$$

Where $n \rightarrow$ order of the system

⇒ The gain matrix is determined through the specs of the observer as:

$\left. \begin{array}{l} * \text{observer speed response} \\ * \text{transient} \\ * \text{settling time} \end{array} \right\} \begin{array}{l} \zeta, \omega_n \rightarrow \text{desired} \\ \text{poles of observer} \end{array}$

→ The gain matrix G is determined by:-

① desired ch. equation for the observer " $\alpha_o(z)$ "

$$\alpha_o(z) = (z-p_1)(z-p_2) \dots (z-p_n) \text{ so } ①$$

where p_1, p_2, \dots, p_n are desired poles

$$\hat{x}(k+1) = (A-GC)\hat{x}(k) + Bu(k) + G y(k)$$

The observer ch-equation:

$$|zI - A + GC| = 0 \rightarrow ②$$

Compare ① & ② to get G

② using Ackermann's method

$$G = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{bmatrix} = \alpha_o(A) M_o^{-1} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$

For $n=2$

$$G = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \alpha_o(A) M_o^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

where $M_o \equiv$ observability matrix

$$\alpha_o(A) = \alpha_o(z) \Big|_{z=A}$$

$\alpha_o(z) \leadsto$ desired ch-equation for observer

Ex $x(k+1) = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} x(k) + \begin{pmatrix} 2 \\ 2 \end{pmatrix} u(k)$

$$y(k) = \begin{pmatrix} 1 & 0 \end{pmatrix} x(k)$$

① Determine the gain matrix K such that desired closed loop poles located at:

$$z_{1,2} = 0.528 \pm j 0.295$$

المطلوب

$$K = \begin{pmatrix} 0.0775 & 0.3945 \end{pmatrix}$$

$$K = \begin{pmatrix} 0 & 1 \end{pmatrix} M_o^{-1} \alpha_c(A)$$

$$\alpha_c(z) = z^2 - 1.056z + 0.366$$

$$\alpha_c(A) = \begin{pmatrix} 0.31 & 1.888 \\ 0 & 0.31 \end{pmatrix}$$

$$M_o^{-1} = \begin{pmatrix} -0.25 & 0.75 \\ 0.25 & -0.25 \end{pmatrix}$$

② Design a full order observer such that $Z=1$ & the time constant of the observer is one half the time constant of the desired poles in ①, $T=1\text{ sec}$

$$\tau_o = \frac{1}{2} \tau_c \rightarrow \text{①?}$$

$$s_{1,2} = -Z\omega_n \pm j\omega_n \sqrt{1-Z^2}$$

للصحيح

$$\hookrightarrow z_{1,2} = r \angle \pm \theta = r \cos \theta \pm j \sin \theta$$

$$r = e^{-Z\omega_n T} = e^{-T/\tau_c}$$

$$\theta = \pm \omega_n T \sqrt{1-Z^2} \quad ; \quad \tau_c = \frac{1}{Z\omega_n}$$

$$r = \sqrt{(0.528)^2 + (0.295)^2} = 0.605$$

$$r = e^{-\frac{T}{\tau_c}} \Rightarrow \ln r = -\frac{T}{\tau_c}$$

$$\ln(0.605) = -\frac{1}{\tau_c} \Rightarrow \tau_c = 2 \text{ sec}$$

$$T_o = \frac{1}{2} T_c$$

$$T_o = \frac{1}{2} \times 2 = 1 \text{ sec}$$

→ The desired poles of the observer:

$$T_o = 1 \text{ sec} \quad \& \quad Z = 1$$

$$T_o = \frac{1}{\omega_n \times Z} \Rightarrow \omega_n = 1$$

$$r = e^{-Z \omega_n T} = e^{-\frac{T}{T_o}} = e^{-1} = 0.3678$$

$$\theta = \omega_n T \sqrt{1 - Z^2} = 0$$

$$Z_{1,2} = r \cos \theta \pm j r \sin \theta = r = 0.3678$$

→ The desired ch. equation for the observer $\alpha_o(z)$:

$$\alpha_o(z) = (z - 0.3678)^2$$

$$\alpha_o(z) = z^2 - 0.7356z + 0.1353$$

$$\alpha_o(A) = A^2 - 0.7356A + 0.1353I$$

$$= \begin{pmatrix} 0.3997 & 2.5288 \\ 0 & 0.3997 \end{pmatrix}$$

$$G = \alpha_o(A) M_o^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$M_o = \begin{pmatrix} I \\ CA \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}$$

$$M_o^{-1} = \begin{pmatrix} 1 & 0 \\ -0.5 & 0.5 \end{pmatrix} \Rightarrow G = \begin{pmatrix} 1.2644 \\ 0.1998 \end{pmatrix}$$

$$\text{Ex 2: } x(k+1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} x(k) + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u(k)$$

$$y(k) = \begin{pmatrix} 1 & 0 \end{pmatrix} x(k)$$

→ Design a Full order observer such that:

a) time constant is 10 sec

b) desired observer poles must be real and equal $\Rightarrow \boxed{Z=1}$

$$T=1 \text{ sec} \quad ; \quad s_{1,2} = -Z\omega_n \pm j\omega_n\sqrt{1-Z^2}$$

*Desired poles of observer

$$r = e^{-\frac{T}{\tau}} = e^{-\frac{1}{10}} = 0.905$$

$$\Theta = \omega_n T \sqrt{1-Z^2} \approx 0 \quad (\text{Real and equal})$$

$\hookrightarrow 1$

$$Z_{1,2} = 0.905$$

\rightarrow The desired ch. eq. for the observer $\alpha_o(z)$:

$$\alpha_o(z) = (z - 0.905)^2 = z^2 - 1.8z + 0.819$$

$$\alpha_o(A) = \begin{pmatrix} 0.009 & 0.19 \\ 0 & 0.009 \end{pmatrix}$$